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Book Review

The Numerical Solution of Ordinary and Partial Differential Equations by Granville Sewell, 3rd Edition, World Scientific, 2015; ISBN: 978-981-4635-080 (hardcover)

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Numerical studies of ODEs and PDEs are useful in physical sciences, diverse areas of engineering, as well as, in computer, biological, economic, and social sciences, to say the least. This approach focuses on the construction and implementation of accurate and efficient numerical algorithms for obtaining approximate solutions. The author of the book is known as an active researcher in this area and this text presents the results of his own investigations into these topics.

The book is organized in six chapters and three appendices. Each chapter begins with an introduction containing the essence of the matter presented later in the text. The chapters contain a large number of instructive exercises, ranging from simple applications of methods and algorithms to generalizations and extensions of the theory.

Chapter 0, *Direct Solution of Linear Systems*, discusses Gaussian elimination method. A terse overview of the algorithms that use direct factorization to solve a various linear system of equations is given. The possible pivoting strategies (partial pivoting, no pivoting) are presented with numerical procedures. Next, the LU factorization method is described. A Fortran programs to solve a banded linear system without and with partial pivoting are presented and discussed also. The parse direct techniques for solving large sparse linear systems, using Gaussian elimination with dissection algorithms, are elucidated briefly.

Chapter 1, *Initial Value Ordinary Differential Equations*, provides an overview of the numerical schemes for the first-order initial value ordinary differential equation problems. First, basic numerical schemes for first-order systems including Euler's method and multistep methods, are recalled. For the sake of completeness, the truncation error analysis is presented. Moreover, the Adams multistep methods and the backward difference methods for stiff problems are discussed. These include: an explicit Adams–Bashforth method, an implicit Adams–Moulton method and a backward difference method. Though these methods are accurate, they are computationally expensive when used to solve high-order initial-value problems. For these problems, the Runge–Kutta methods are preferable. These methods are the topic of the last section.

Chapter 2, *The Initial Value Diffusion Problem*, is concerned with the numerical treatment of partial differential equation that models diffusion (or heat conduction). The brief derivation of this equation, based on the conservation principles as applied to an arbitrary control volume, is presented first. Besides the classical forward difference approximation, the implicit methods are presented and discussed. Among these, the Crank–Nicolson method is of particular importance. Numerical examples illustrating the behavior of these methods, including the exact solution of 1D diffusion problem, two-dimensional diffusion equation and nonlinear diffusion–reaction problem are provided and analyzed.

Chapter 3, *The Initial Value Transport and Wave Problems*, deals with the analysis and numerical simulation of diffusion-convection and wave propagation problems described by systems of hyperbolic

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partial differential equations. Illustrative examples include the one-dimensional version of the transport equation, the vibrating string problems and the system of equations governing displacements in an elastic body. Two numerical methods for the transport problem are discussed and analyzed, namely, the explicit methods, and the method of characteristics. Then, an explicit scheme for the 1D vibrating string problem is presented. To illustrate the accuracy and efficiency of the method, a 2D damped wave problem in a triangular region is studied.

Chapter 4, *Boundary Value Problems*, focuses on the steady-state (or time-independent) phenomena. The concept of eigenvalue in connection with boundary value problems is elucidated. A brief discussion of simple finite difference methods for linear, 1D diffusion-convection problem is presented, together with the nonlinear and singular examples. The ensuing section explains the shooting technique for linear problems and its variant, multiple shooting. The iterative methods applied to the multidimensional problems are discussed in the following sections. These include: successive overrelaxation with the examples, and the conjugate-gradient iterative method. Moreover, the methods to solve systems of differential equations are mentioned briefly. Finally, the 1D eigenvalue problem is formulated and the inverse power method is used to determine the dominant eigenvalue of the appropriate matrix.

Chapter 5, *The Finite Element Method*, concentrates on the finite-elements schemes. A general introduction to the Galerkin methods for solving 3D boundary value problem is presented first. The list of questions approximated and analyzed using Galerkin approach includes, among others, examples using piecewise linear trial functions and cubic Hermite trial functions. The method of collocation, in which the solution is expanded as a linear combination of polynomial basis function, is demonstrated. As an example of numerical performance of the method, numerical solution to generic singular problem is showed. Linear triangular elements applied to solve a 2D linear problem are discussed also. The use of the continuous-time Galerkin scheme to solve time-dependent problems is explained. The computational examples concern 1D and 2D problems. Finally, finite elements methods for the linear eigenvalue problems are considered with commentaries.

Appendix A describes PDE2D 9.4, the program intended to solution of quite general nonlinear, time-dependent, steady-state and eigenvalue systems of partial differential equations, in 1D intervals, general 2D regions and a wide range of simple 3D domains. The full program is commercial, but, fortunately, the author makes accessible the free versions available for small to moderate size computational problems for several well-known operating systems (http://www.pde2d.com). Appendix B gives some facts about the Fourier stability method. Appendix C contains all of the Fortran90 codes given in chapters 0–5, in the form of the Matlab m-files (see http://www.math.utep.edu/Faculty/sewell/odes_pdes/). At last, Appendix D includes answers to selected exercises which are presented in the book.

This book is very well written and it is relatively easy to read. The presentation is clear and straightforward but quite rigorous. This book is suitable for a course on the numerical solution of ODEs and PDEs problems, designed for senior level undergraduate or beginning level graduate students. The reader is assumed to have a basic knowledge of multivariate calculus, linear algebra and elements of numerical analysis. The numerical techniques for solving problems presented in the book may also be useful for experienced researchers and practitioners both from universities or industry.